Gwynedd Mercy Academy High School

MATH 0438: AP® Calculus BC

Summer Assignment

Overview: Welcome to AP® Calculus BC! As with most AP® courses, the breadth and depth of the material covered on the AP® Calculus BC exam tends to exceed the amount of class time available. To that end, I would like you to get a head start on the content (specifically, the review of the AP® Calculus AB subtopics) to ensure we have enough time to learn the necessary material and review for the AP® exam next May. This assignment will be considered your first graded assignment for AP® Calculus BC. If you have any questions, contact Mr. Straniero at dstraniero@gmahs.org.

Part I: Differential Calculus Essentials

1. $\lim_{x\to 0} \pi^2 =$

2. $\lim_{x\to\infty}\left(\frac{10x^2+25x+1}{x^4-8}\right)=$

3. $\lim_{x\to\infty}\left(\frac{\sqrt{5x^4+2x}}{x^2}\right)=$

4. $\lim_{x\to 0^-}\left(\frac{x}{|x|}\right)=$

5. $\lim_{x\to 7}\frac{x}{(x-7)^2}=$

6. Find $\lim_{x\to 0}\frac{\sin 3x}{\sin 8x}$.

7. Find $\lim_{x\to 0}\frac{x^2\sin x}{1-\cos^2 x}$.

8. Find $\lim_{h\to 0}\frac{\sin(x+h)-\sin x}{h}$.

9. Is the function $f(x)=\begin{cases}5x+7, & x<3\\ 7x+1, & x>3\end{cases}$ continuous at $x = 3$?

10. For what value(s) of k is the function $f(x) = \begin{cases}-6x-12, & x<-3\\ k^2-5k, & x=-3\\ 6, & x>-3\end{cases}$ continuous at $x = -3$?

11. Find the derivative of $f(x) = 2x^2$ at $x = 5$.

For problems 12–19, find the derivative.

12. $f(x) = x^4$

13. $f(x) = \cos x$

14. $f(x)= \dfrac{1}{x^2}$

15. $8x^{10}$

16. $\dfrac{1}{a}\left(\dfrac{1}{b}x^2 - \dfrac{2}{a}x - \dfrac{d}{x}\right)$

17. $\sqrt{x} + \dfrac{1}{x^3}$

18. $(x^2 + 8x - 4)(2x^{-2} + x^{-4})$

19. $ax^5 + bx^4 + cx^3 + dx^2 + ex + f$

20. Find $f'(x)$ if $f(x) = (x+1)^{10}$.

21. Find $f'(x)$ if $f(x) = \dfrac{4x^8 - \sqrt{x}}{8x^4}$.

22. Find $f'(x)$ at $x = 1$ if $f(x) = \left[\dfrac{x-\sqrt{x}}{x+\sqrt{x}}\right]^2$.

23. Find $f'(x)$ at $x = 1$ if $f(x) = \dfrac{x}{(1+x^2)^2}$.

24. Find $\dfrac{du}{dv}$ at $v = 2$ if $u = \sqrt{x^3 + x^2}$ and $x = \dfrac{1}{v}$.

25. Find $\dfrac{dy}{dx}$ if $y = \cot 4x$.

26. Find $\frac{dy}{dx}$ if $y = 2 \sin 3x \cos 4x$.

27. Find $\frac{dr}{d\theta}$ if $r = \sec\theta \tan 2\theta$.

28. Find $\frac{dy}{dx}$ if $y = \sin\left(\cos\left(\sqrt{x}\right)\right)$.

29. Find $\frac{dy}{dx}$ if $\cos y - \sin x = \sin y - \cos x$.

30. Find $\frac{dy}{dx}$ if $x^{\frac{1}{2}} + y^{\frac{1}{2}} = 2y^2$ at (1, 1).

Part II: Differential Calculus Applications

1. Find the equation of the normal to the graph of $y = \sqrt{8x}$ at $x = 2$.

2. Find the equation of the tangent to the graph of $y = 4 - 3x - x^2$ at (0, 4).

3. Find the equation of the tangent to the graph of $y = (x^2 + 4x + 4)^2$ at $x = -2$.

4. Find the values of c that satisfy the MVTD for $f(x) = x^3 + 12x^2 + 7x$ on the interval $[-4, 4]$.

5. Find the values of c that satisfy Rolle's Theorem for $f(x) = x^3 - x$ on the interval $[-1, 1]$.

6. A computer company determines that its profit equation (in millions of dollars) is given by $P = x^3 - 48x^2 + 720x - 1{,}000$, where x is the number of thousands of units of software sold and $0 \le x \le 40$. Optimize the manufacturer's profit.

7. Find the dimensions of the rectangle with maximum area that can be inscribed in a circle of radius 10.

8. Find the coordinates of any maxima/minima and points of inflection of the following function. Then sketch the graph of the function.

$$y = \frac{x^4}{4} - 2x^2$$

9. Find the coordinates of any maxima/minima and points of inflection of the following function. Then sketch the graph of the function.

$$y = \frac{3x^2}{x^2 - 4}$$

10. A cylindrical tank with a radius of 6 meters is filling with fluid at a rate of 108π m^3/s. How fast is the height increasing?

11. The voltage, V, in an electrical circuit is related to the current, I, and the resistance, R, by the equation $V = IR$. The current is decreasing at -4 amps/s as the resistance increases at 20 ohms/s. How fast is the voltage changing when the voltage is 100 volts and the current is 20 amps?

12. If the position function of a particle is $x(t) = \dfrac{t}{t^2 + 9}$, $t > 0$, find when the particle is changing direction.

13. If the position function of a particle is $x(t) = \sin^2 2t$, $t > 0$, find the distance that the particle travels from $t = 0$ to $t = 2$.

For questions 14–20, find the derivative of each function.

14. $f(x) = x\ln\cos 3x - x^3$

15. $f(x) = \dfrac{e^{\tan 4x}}{4x}$

16. $f(x) = \log_6(3x\tan x)$

17. $f(x) = \ln x \log x$

18. $f(x) = 5^{\tan x}$

19. $f(x) = x^7 - 2x^5 + 2x^3$ at $f(x) = 1$

20. $y = x^{\frac{1}{3}} + x^{\frac{1}{5}}$ at $y = 2$

21. Approximate $(9.99)^3$.

22. A side of an equilateral triangle is measured to be 10 cm. Estimate the change in the area of the triangle when the side shrinks to 9.8 cm.

23. $\lim\limits_{x \to 0} \dfrac{\sqrt{5x+25}-5}{x} =$

24. $\lim\limits_{x \to 0} \dfrac{\theta - \sin\theta\cos\theta}{\tan\theta - 0} =$

Part III: Integral Calculus Essentials

1. Evaluate $\int \frac{x^5+7}{x^2}\,dx$.

2. Evaluate $\int (1+x^2)(x-2)\,dx$.

3. Evaluate $\int (\cos x - 5\sin x)\,dx$.

4. Evaluate $\int \frac{\sin x}{\cos^2 x}\,dx$.

5. Evaluate $\int (\tan^2 x)\,dx$.

6. Evaluate $\int \frac{dx}{(x-1)^2}$.

7. Evaluate $\int \frac{\sin 2x}{(1-\cos 2x)^3}\,dx$.

8. Evaluate $\int_{-4}^{4} |x|\,dx$.

9. Find the area under the curve $y = 2 + x^3$ from $x = 0$ to $x = 3$ using $n = 6$ right-endpoint rectangles.

10. Find the average value of $f(x) = \sqrt{1-x}$ on the interval $[-1, 1]$.

11. Find $\frac{d}{dx}\int_0^{x^2} |t|\,dt$.

12. Evaluate $\int \frac{1}{x}\cos(\ln x)\,dx$.

13. Evaluate $\int e^x \cos(2+e^x)\,dx$.

14. Evaluate $\int \frac{e^{3x}\,dx}{1+e^{6x}}$.

15. If $f(\theta) = \sin^{-1}(4\theta)$, find $f'\left(\frac{1}{8}\right)$.

Part IV: Integral Calculus Applications

1. Find the area of the region between the curve $y = x^3$ and the curve $y = 3x^2 - 4$.

2. Find the area of the region between the curve $x = y^3 - y^2$ and the line $x = 2y$.

3. Find the volume of the solid that results when the region bounded by $y = x^3$, $x = 2$, and the x-axis is revolved around the line $x = 2$.

4. Find the volume of the solid that results when the region bounded by $y^2 = 8x$ and $x = 2$ is revolved around the line $x = 4$.

5. If $\frac{dy}{dx} = \frac{1}{y + x^2 y}$ and $y(0) = 2$, find an equation for y in terms of x.

6. The rate of growth of the volume of a sphere is proportional to its volume. If the volume of the sphere is initially 36π ft^3, and expands to 90π ft^3 after 1 second, find the volume of the sphere after 3 seconds.

7. Sketch the slope field for $\frac{dy}{dx} = \frac{x}{y}$.